The Cosmic Gamma-Ray Spectrum From Secondary Particle Production Outside the Galaxy

In chapters 5 and 6 we made use of recent accelerator and cosmic-ray data to determine the details of the cosmic γ-ray spectrum from the secondary particles produced by cosmic-ray collisions in the galaxy. Here, we will make use of the cosmological formulas derived in chapter 9 in order to determine the cosmic γ-ray spectrum from secondary particles produced by cosmic-ray collisions in extragalactic space. This spectrum will differ from the galactic (or local) γ-ray spectrum because most of the generating collisions take place at large distances where we are looking back to a time when the universe was more compact and collisions were more frequent. These "early" γ-rays will be of lower energy due to the progressive redshift of the general cosmic expansion. Using the formulas derived in chapter 9, we take account of time-dilatation, volume-diminution, redshift, and curvature effects which become important at large redshifts.

In chapter 9, we obtained the important relation between redshift and radius of the universe, given by equation (9–15)

$$\frac{R(t_r)}{R(t_e)} = 1+z$$

where t_e is the time when the γ-ray was emitted and t_r is the time when the γ-ray is received (detected).

It follows from equation (9–15) that in a universe where most of the energy density is in the form of matter,

$$\frac{n(t_e)}{n(t_r)} = (1+z)^3 \qquad (10\text{–}1)$$

$$\frac{T_\gamma(t_e)}{T_\gamma(t_r)} = (1+z) \qquad (10\text{–}2)$$

and

$$\frac{n_\gamma(t_e)}{n_\gamma(t_r)} = (1+z)^3 \qquad (10\text{–}3)$$

where $n(t)$, $T_\gamma(t)$, and $n_\gamma(t)$ are the average particle density of matter, and the temperature and photon density of cosmic blackbody radiation in the universe. Let $G_g(E_\gamma)$ be the γ-ray spectrum generated by the galactic cosmic-ray spectrum, $I_g(E_p)$, in traveling a unit particle length (1 cm^{-2}) through the intergalactic medium. (This spectrum is the same as the quantity $I(E_\gamma)/\langle nL\rangle$ calculated in chs. 5 and 6.)

We now assume that some ubiquitous generating mechanism causes cosmic rays to be produced throughout the universe with the same power law as observed at the Earth, so that the metagalactic cosmic-ray spectrum differs only in absolute intensity from the galactic cosmic-ray spectrum. It follows that the form of the cosmic γ-ray spectrum anywhere in the metagalaxy, when observed in the comoving frame at that point, will be the same as the form of $G_g(E_\gamma)$. We may then write down an expression for the integrated metagalactic γ-ray flux in any direction as

$$I_\pi(E_\gamma) = \int_0^{z_{\max}} dz\, n(z) \frac{I_c(z)}{I_g} \frac{G_g[(1+z)E_\gamma]}{(1+z)^2} e^{-\tau(E_\gamma, z)} \frac{dl}{dz} \qquad (10\text{–}4)$$

with the derivation of equation (10–4) being similar to the derivation of equation (9–22) in the previous chapter. Here we have used the symbol I_g to represent the total intensity of galactic cosmic rays of energies above the pion-production threshold in order to normalize the generating spectrum G_g and to relate $I_\pi(E_\gamma)$ to the extragalactic cosmic-ray intensity $I_c(z)$. We have also included in equation (10–4) an added exponential factor to account for absorption of γ-rays, which may become important at high redshifts under certain conditions which we will discuss later. The following evaluation of equation (10–4) is based on a calculation by the author (Stecker, 1969c).

We will assume, for the purposes of calculating $I_\pi(E_\gamma)$, that at some early epoch, corresponding to $z \geq z_{\max}$, conditions were unsuitable for the acceleration of cosmic rays. We will consider $z_{\max}$ to correspond to the epoch of galaxy formation and consider two possible models for the origin of a metagalactic cosmic-ray flux. For model I, we will assume that the extragalactic cosmic-ray flux arises through leakage from the halos of radio galaxies from $z = z_{\max}$ to $z = 0$. For model II, we will assume that this flux was created primarily in a burst at the time of galaxy formation. For $z_{\max}$, we will also consider two extremes. One extreme is $z_{\max} = 10^2$, which corresponds to the earliest epoch when galaxy formation could probably occur. At $z = 10^3$, the blackbody temperature of the universe was of the order of 10^3 to 10^4 K, cool enough for ionized hydrogen to combine to form a neutral gas. According to Peebles (1965), $z = 10^3$ also corresponds to the epoch when gas clouds may have begun to form gravitationally bound systems.

The other extreme for z_{max} which we may consider corresponds to the highest redshift yet observed for a quasar; viz, 2.2.[1] This is, of course, an extreme which is limited by technique rather than by any physical criteria, and it is included mainly for purposes of discussion. We will also consider various intermediate values for z_{max} of 4, 9, and 40. (Doroshkevich, Zeldovich, and Novikov (1967) suggest that galaxy formation took place at $z = 10$ to 20, whereas Weymann (1967) suggests $z = 10^2$.)

It is important to note here that the upper limit z_{max} may be effectively restricted, not by the epoch of galaxy formation, but by attenuation of the metagalactic cosmic-ray flux due to the collisions themselves. The cross section for inelastic cosmic-ray p–p collisions is of the order of 30 mb. Therefore, the lifetime of the metagalactic cosmic rays against collisional losses is given by

$$T_c = \frac{1}{n\sigma c} = \frac{1}{n_0 \sigma c (1+z)^3}$$

$$\simeq 10^{15} n_0^{-1} (1+z)^{-3} \text{ s} \qquad (10\text{–}5)$$

The lifetime of the universe at a redshift z is

$$T_u \simeq 3 \times 10^{17} (1+z)^{-3/2} \text{ s} \qquad (10\text{–}6)$$

for an Einstein-de Sitter model, where $n_0 \simeq 10^{-5}$ cm^3.

Cosmic rays cannot accumulate in the metagalaxy if the ratio $T_c/T_u < 1$. Therefore, the condition $T_c/T_u = 1$ defines a critical value for an Einstein-de Sitter model of $z_{max} \sim 10^2$ beyond which a further buildup of metagalactic cosmic rays cannot occur. With these limitations on z in mind, we will now consider the various ideal models for describing the metagalactic cosmic-ray flux.

For model I, we assume a constant leakage rate so that the total number of cosmic rays in the metagalaxy is proportional to the time elapsed since galaxy formation. It follows from equation (10–6) that for an Einstein-de Sitter model this time is given by

$$\tau_g \simeq 10^{10} [(1+z)^{-3/2} - (1+z_{max})^{-3/2}] \quad \text{years} \qquad (10\text{–}7)$$

The cosmic-ray density will then increase with redshift according to the relation

$$\frac{I'(z)}{I_g} \sim (1+z)^3 [(1+z)^{-3/2} - (1+z_{max})^{-3/2}] \qquad (10\text{–}8)$$

[1] This number has since grown.

However, the cosmic rays which produce the neutral pions necessary for γ-ray production are only those above a threshold kinetic energy $E_{th} - M_p$ of about 300 MeV. We must therefore determine

$$I(z) = I'(E > E_{th}|z) \tag{10-9}$$

For a power-law cosmic-ray spectrum of the form

$$I(>E) \sim E^{-1.5} \tag{10-10}$$

as is approximately valid in the energy region where most pions are produced (see ch. 5), it follows from the redshift relation that

$$\begin{aligned} I(E > E_{th}|z) &= I'\left(E > \frac{E_{th}}{(1+z)}\right) \\ &= I'(z)\left[\frac{1+z}{1+z_{max}}\right]^{1.5} \end{aligned} \tag{10-11}$$

so that we must use an effective flux of

$$\frac{I(z)}{I_g} \sim (1+z)^3 \left[\frac{1+z}{1+z_{max}}\right]^{1.5} [(1+z)^{-3/2} - (1+z_{max})^{-3/2}] \tag{10-12}$$

For model II, we assume that the metagalactic cosmic rays were created in a burst at the time of galaxy formation. We thus find for this model that

$$\frac{I(z)}{I_g} \sim (1+z)^3 \left[\frac{1+z}{1+z_{max}}\right]^{1.5} \tag{10-13}$$

Using the models defined by equations (10–12) and (10–13), together

TABLE 10–1.—*Value of* I_0/I_g *for* $n_0 = 10^{-5}$ cm^{-3}

[Stecker, 1969c]

Model	z_{max}				
	2.2	4	9	40	100
Constant-leakage model	1.8×10^{-1}	1.3×10^{-1}	7.0×10^{-2}	3.2×10^{-2}	2.8×10^{-2}
Burst model	2.6×10^{-2}	9.0×10^{-3}	2.2×10^{-3}	3.0×10^{-4}	1.1×10^{-4}

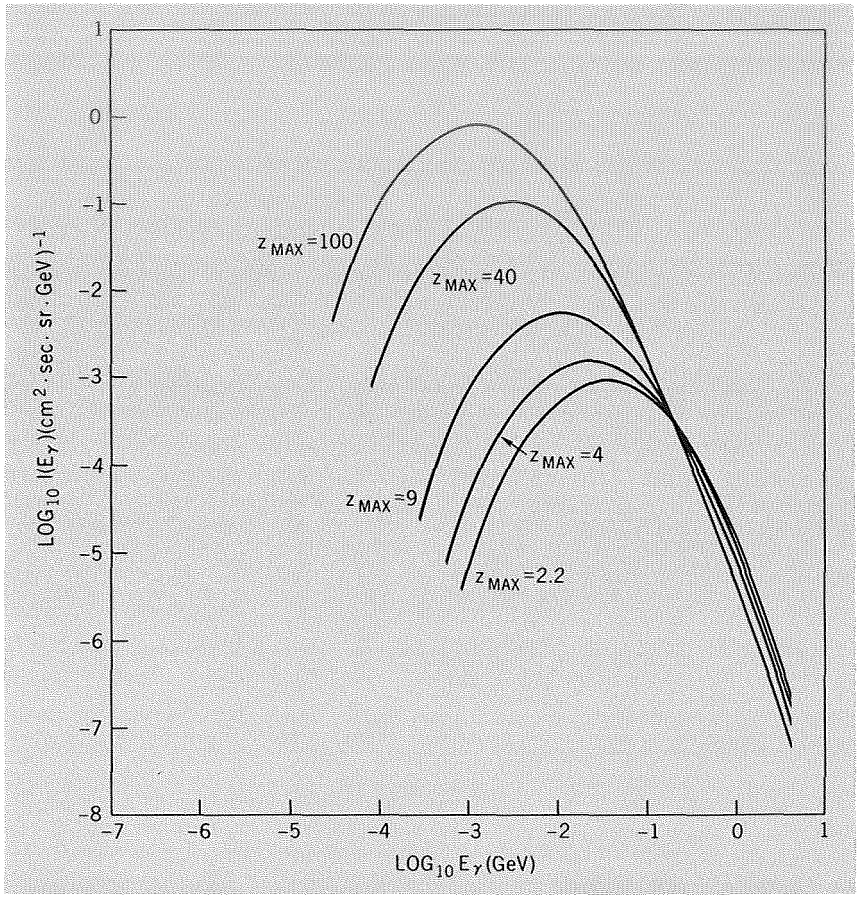

FIGURE 10–1.—Differential spectrum for leakage model (model I) (Stecker, 1969c).

with the value for dl/dz given by equation (9–30), Stecker (1969c) used equation (10–4) to calculate the extragalactic γ-ray spectra produced by cosmic-ray models I and II. These results were normalized by requiring the integral γ-ray spectrum above 100 MeV to be equal to 1.1×10^{-4} cm^{-2}-s^{-1}-sr^{-1}, according to the results of Clark et al. (1968) measured by the detector aboard the OSO–3 satellite.[2] As has been noted previously, such a normalization makes possible the determination of upper limits on the value I_0/I_g; i.e., the present metagalactic intensity of cosmic-ray nucleons. These upper limits are given in table 10–1.

The γ-ray fluxes thus calculated are given in figures 10–1 to 10–4. Figure 10–5 shows the γ-ray flux expected from the galactic halo in the

[2] This value has now been determined by Clark et al. (1970) to be approximately a factor of 2 to 4 too high.

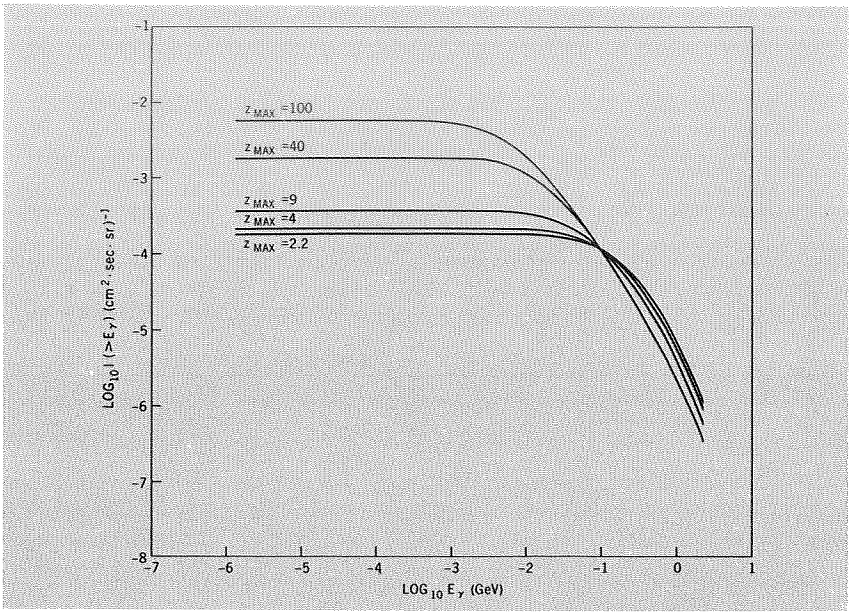

FIGURE 10–2.—Integral spectrum for leakage model (model I) (Stecker, 1969c).

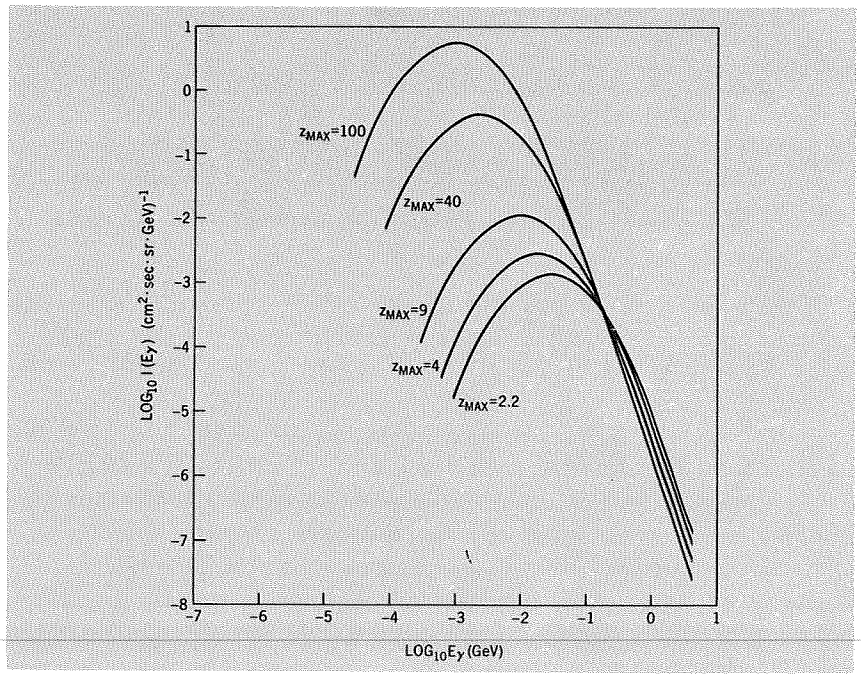

FIGURE 10–3.—Differential spectrum for burst model (model II) (Stecker, 1969c).

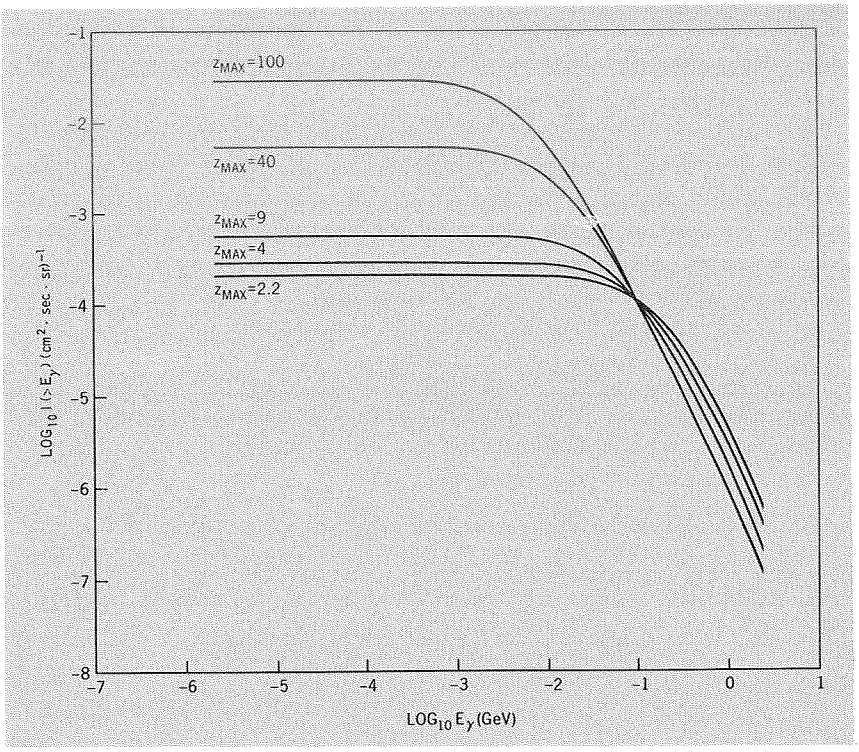

FIGURE 10–4.—Integral spectrum for burst model (model II) (Stecker, 1969c).

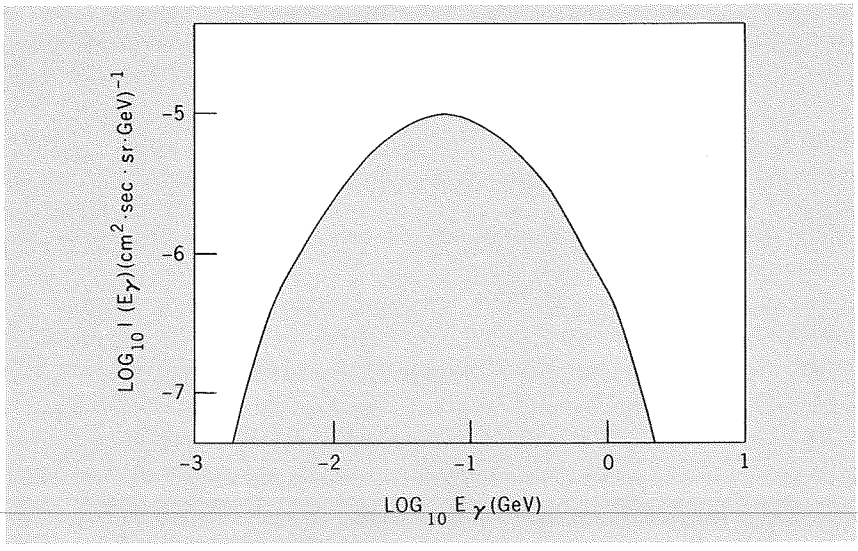

FIGURE 10–5.—Differential spectrum for the galactic halo ($\langle nL \rangle = 3 \times 10^{20}$ cm^{-2}).

direction of the pole, based on the results of chapters 5 and 6 and taking $\langle nL \rangle = 3 \times 10^{20}$ cm$^{-2/3}$. It can be seen that the local γ-ray flux from the galactic halo will not explain the data of Clark et al. and should be unimportant compared to the extragalactic flux. The extragalactic γ-ray spectra tend to peak near $7 \times 10^{-2}/(1+z_{max})$ GeV, being weighted toward higher redshifts by the effect of greater densities at earlier epochs. Because of the density effect, a cosmic-ray burst at large redshifts is much more effective in producing γ-rays than a continuous production of the same number of cosmic rays.

---

[3] Thus $I_{pole}(E_\gamma) = 3 \times 10^{20}\, G_g(E_\gamma)$.